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Stability of Isothermal Spinning of Viscoelastic Fluids

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Abstract–The stability of isothermal spinning of viscoelastic fluids which have strain-rate dependent relaxation time has been investigated using the linear stability analysis method. The instability known as draw resonance of the system was found to be dependent upon the material functions of the fluids like fluid relaxation time and the strain-rate dependency of the relaxation time as well as upon the draw-down ratio of the process. Utilizing the fundamental physics of the system characterized by the traveling kinematic waves, we also have developed a simple, approximate method for determining this draw resonance instability ; it requires only the steady state velocity solutions of the system, in contrast to the exact stability analysis method which requires solving the transient equations. The stability curves produced by this simple, fast method agree well with those by the exact stability method, proving the utility of the method. The stability of other extensional deformation processes such as film casting and film blowing can also be analyzed using the method developed in this study.

Key words : Draw Resonance, Linear Stability Analysis, Maxwell Fluids, Strain-Rate Dependent Relaxation Time, Throughput Waves, Traveling Velocity

INTRODUCTION

The stability of polymer processing is very important in many respects. First, it is one of the most important subjects to the people who run the polymers manufacturing facilities, because along with the sensitivity of the process to outside disturbances it is always related to the safety, productivity, product quality, and ultimately to the profitability issue of the concern. Second, stability is always of the first interest to theoreticians who study the fundamental aspects of nonlinear dynamic systems like existence and uniqueness of the solutions, possibility of oscillation, bifurcation and chaos, etc. Third, due to the intricate nature of the polymer materials in their structure, and their flow and deformation behavior, the dynamics of most polymer processing is extremely complicated [Petrie and Denn, 1976].

Thus the stability of continuous processes of polymer processing like fiber spinning, film casting, film blowing, calendering, pultrusion, etc. has long been an exciting subject for many researchers around the world. But it was 1960s when the first attempt was made regarding this stability issue in the fiber spinning field [Kase and Matsuo, 1965; Matovich and Pearson, 1969].

Then it became immediately clear that even in this seemingly simple fiber spinning, the complexity of the dynamics involved is immense. The three dimensional nature of dynamics, the phase changes occurring inside the fiber, the difficulty in modeling the stress variables in the amorphous/crystalline structure, complex heat transfer with the heat of crystallization, the inertia effect in high speed spinning, viscoelasticity of polymer melts, nonlinear constitutive equations, etc. are such examples, to name just a few [Avenas et al., 1975; Tsou and Bogue, 1985; Ziabicki and Kawai, 1985; Spruiell et al., 1991].

Against the backdrop of all these fiber spinning details, the necessity of a stability study of spinning has remained strong. This is because of two main reasons. One is that as the knowledge level of spinning advances, the fundamental understanding of the process dynamics, such as its stability, becomes even more important. The other is the fact that other polymer processing like film casting and film blowing, where extensional deformation dominates, possesses basically the same dynamics as spinning, so that extensional phenomena like draw resonance, which is characterized by a sustained oscillation in the fiber radius and spinline tension, is equally important in all these processes [Fisher and Denn, 1976; White and Ide, 1978; Hyun, 1978; Cain and Denn, 1988; Anturkar and Co., 1988; Kim et al., 1996].

In this study, the draw resonance stability of the isothermal spinning of convected Maxwell fluids which possess strain-rate dependent relaxation time has been studied. First, the linear stability analysis method is employed to study the characteristics of the stability of the system including the effects of material functions like fluid relaxation time (elasticity) and the strain-rate dependency of the relaxation time. Next, a simple, approximate method for determining the same stability has also been derived based on the fundamental physics of spinning i.e., the fact that spinning is a hyperbolic process possessing various kinematic waves traveling the spinline including the throughput waves. The stability curves obtained by this approximate method are seen to agree well with the exact ones generated by the linear stability method, proving the utility of the method as a useful analysis tool in extensional deformation processes.

PROCEDURE OF LINEAR STABILITY ANALYSIS

The governing equations of the isothermal spinning of con-

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vected Maxwell fluids in this study are as follows [Avenas et al., 1975; Ide and White, 1977; Beris and Liu, 1988]; a schematic drawing of the melt spinning process is shown in Fig. 1.

Equation of continuity:

$$\frac{\partial A}{\partial t'}+\frac{\partial (AV)}{\partial z}=0 \quad (1)$$

Equation of motion:

$$\frac{\partial}{\partial z}[A(\sigma_{zz}-\sigma_{rr})]=0 \quad (2)$$

Constitutive equation:

$$\sigma_{zz}+\lambda\left(\frac{\partial \sigma_{zz}}{\partial t'}+V\frac{\partial \sigma_{zz}}{\partial z}-2\sigma_{zz}\frac{\partial V}{\partial z}\right)=2G\lambda\frac{\partial V}{\partial z} \quad (3)$$

$$\sigma_{rr}+\lambda\left(\frac{\partial \sigma_{rr}}{\partial t'}+V\frac{\partial \sigma_{rr}}{\partial z}+\sigma_{rr}\frac{\partial V}{\partial z}\right)=-G\lambda\frac{\partial V}{\partial z} \quad (4)$$

Strain-rate dependent material relaxation time [Ide and White, 1977]:

$$\lambda=\frac{\lambda_o}{1+\bar{a}\sqrt{3}\lambda_o\frac{\partial V}{\partial z}} \quad (5)$$

Boundary conditions:

$$A=A_o,\ V=V_o,\ \sigma_{zz}=\sigma_o \text{ at } z=0 \text{ for all } t' \quad (6)$$

$$V=V_L=rV_o \qquad \text{at } z=L \text{ for all } t' \quad (7)$$

(The notations appearing here are given in the Nomenclature.)

In the above, the following assumptions have been incorporated. First, the whole model is in one-dimensional format, meaning that the distance coordinate is the only independent space variable. Second, all the secondary forces on the spinline, i.e., inertia, gravity, air drag, and surface tension, are neglected. Third, the origin of the space coordinate is chosen at the point of extrudate swell, meaning that all the pre-spinneret deformation history of the fluid is not included in the model.

All these assumptions were adopted in order to simplify the model and to focus on the extensional deformation which constitutes dominant dynamics in spinning.

Parameter $\bar{a}$ of Eq. (5) represents the strain-rate dependency of material relaxation time which was first introduced by Ide and White [1977] and then extensively used by Minoshima and White [1986] in both theoretical and experimental analyses of various extensional deformation processes.

In order to qualitatively illustrate the draw resonance phenomenon, Fig. 2 is provided here showing the transient behavior of the cross-sectional area at the take-up at three different values of the draw-down ratio. When the draw-down ratio, r, is larger than its critical value, i.e., $r \geq r_C$, the draw resonance is clearly established as steady oscillations with distinct periods and its severity increases with increasing r, whereas if r is smaller than r_C, the system is stable with all disturbances dying out with time.

Now the usual steps of linear stability analysis are followed. First, the nondimensionalization of the above governing equations is in order.

Equation of continuity:

$$\frac{\partial a}{\partial t}+\frac{\partial (av)}{\partial x}=0 \quad (8)$$

where, $t=t'V_o/L$, $x=z/L$, $a=A/A_o$, $v=V/V_o$ (9)

Equation of motion:

$$\frac{\partial}{\partial x}[a(\tau_{xx}-\tau_{rr})]=0 \quad (10)$$

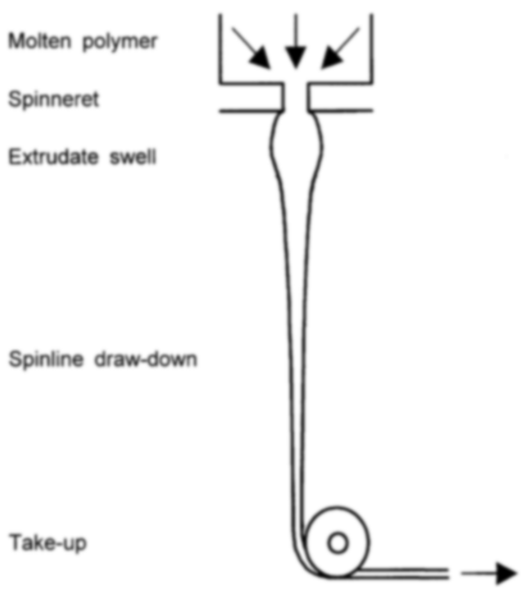

Fig. 1. Schematic diagram of the melt spinning process.

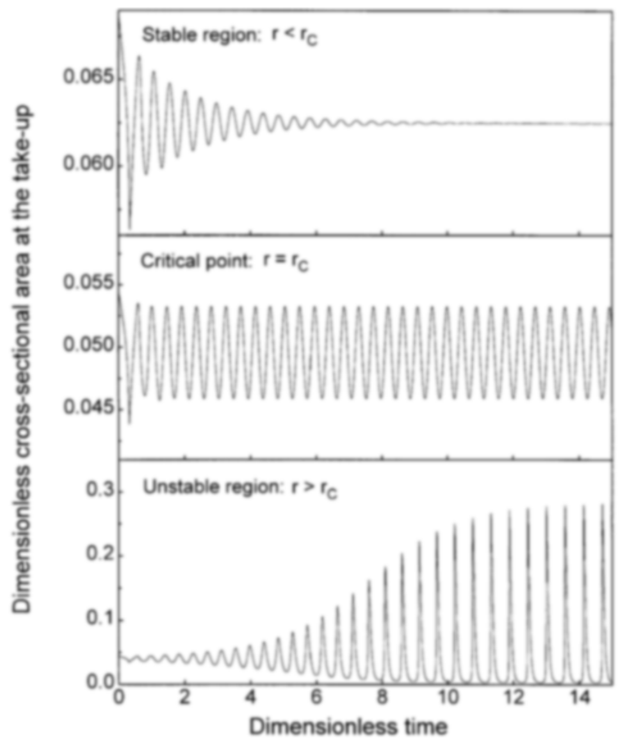

Fig. 2. Transient response of the dimensionless area at the take-up.

where, $\tau_{xx}=\sigma_{zz}A_o/F$, $\tau_{rr}=\sigma_{rr}A_o/F$ (11)

Constitutive equation :

$$\tau_{xx}\left(1+\bar{a}\sqrt{3}De\frac{\partial v}{\partial x}\right)+De\left(\frac{\partial \tau_{xx}}{\partial t}+v\frac{\partial \tau_{xx}}{\partial x}-2\tau_{xx}\frac{\partial v}{\partial x}\right)=2g\frac{\partial v}{\partial x} \tag{12}$$

$$\tau_{rr}\left(1+\bar{a}\sqrt{3}De\frac{\partial v}{\partial x}\right)+De\left(\frac{\partial \tau_{rr}}{\partial t}+v\frac{\partial \tau_{rr}}{\partial x}+\tau_{rr}\frac{\partial v}{\partial x}\right)=-g\frac{\partial v}{\partial x} \tag{13}$$

where, $De=\lambda_o V_o/L$, $g=G\lambda_o A_o/F$ (14)

Boundary conditions :

$$a=1,\ v=1,\ \tau_{xx}=\tau_o \text{ at } x=0 \text{ for all } t \tag{15}$$

$$v=r \qquad \text{at } x=1 \text{ for all } t \tag{16}$$

where τ_o is a specified value of stress at the spinneret which turns out to be relatively unimportant in determining the dynamics of spinning process.

Next these dimensionless equations are linearized and then the perturbations are introduced to the dependent variables as follows.

$$a(t,x)=a_s(x)+\alpha(x)e^{\Omega t} \tag{17}$$

$$v(t,x)=v_s(x)+\beta(x)e^{\Omega t} \tag{18}$$

$$\tau_{xx}(t,x)=\tau_{xx,s}(x)+\gamma(x)e^{\Omega t} \tag{19}$$

$$\tau_{xx}(t,x)-\tau_{rr}(t,x)=\tau_{xx,s}(x)-\tau_{rr,s}(x)+\delta(x)e^{\Omega t}=v_s(x)+\delta(x)e^{\Omega t} \tag{20}$$

The subscripts s indicates steady state, α, β, γ and δ are the perturbed quantities and Ω is a complex eigenvalue that accounts for the growth rate of the perturbation.

Insertion of these perturbed variables into the above dimensionless linearized equations produces the following linear equations.

Equation of continuity :

$$\Omega\alpha=(-v_s')\alpha-(v_s)\alpha'+\left(\frac{v_s'}{v_s^2}\right)\beta-\left(\frac{1}{v_s}\right)\beta' \tag{21}$$

Equation of motion :

$$(v_s')\alpha+(v_s)\alpha'=\left(\frac{v_s'}{v_s^2}\right)\delta-\left(\frac{1}{v_s}\right)\delta' \tag{22}$$

Constitutive equation :

$$\Omega\gamma=\left((2-\bar{a}\sqrt{3})v_s'-\frac{1}{De}\right)\gamma-(v_s)\gamma'-(\tau_{xx,s}')\beta+\left(\frac{2g}{De}+(2-\bar{a}\sqrt{3})\tau_{xx,s}\right)\beta' \tag{23}$$

$$\Omega\delta=\left(-(1+\bar{a}\sqrt{3})v_s'-\frac{1}{De}\right)\delta-(v_s)\delta'+(3v_s')\gamma-(v_s')\beta +\left(3\tau_{xx,s}+\frac{3g}{De}-(1+\bar{a}\sqrt{3})v_s\right)\beta' \tag{24}$$

Boundary conditions :

$$\alpha(0)=\beta(0)=\beta(1)=\gamma(0)=0 \tag{25}$$

In the above equations, superscript ′ denotes $\partial/\partial x$.

Discretizing and rearranging the above equations, the following algebraic linear matrix equation is obtained.

$$\Omega\underline{y}=\underline{\underline{A}}\,\underline{y} \tag{26}$$

where, $\underline{y}=[\gamma_1,\gamma_2,\cdots,\gamma_N,\delta_0,\delta_1,\delta_2,\cdots,\delta_N]^T$

$\underline{\underline{A}}$ is (2N+1, 2N+1) matrix whose components are determined from the algebraic manipulations of Eq. (21)-(25) and N is the number of mesh points in the discretized spinning distance from spinneret to take-up.

Given all the parameters and boundary conditions, the eigenvalues of Eq. (26) can be readily obtained. Table 1 shows such results of the real and imaginary parts of the eigenvalue when the values of the draw-down ratio and material functions are given. Here it is immediately noticed that the values of critical draw-down ratio at the onset of draw resonance are readily obtained by finding their values, which makes the real parts of the largest eigenvalue equal to zero. The stability curves separating the stable and unstable regions in the parameter space are also readily obtained from the data in Table 1.

Fig. 3 shows such stability curves portraying the effect of material functions and process conditions. Specifically, the Deborah number, De, representing the dimensionless material relaxation time and the draw-down ratio, r, are describing the stability regions here along with the strain-rate dependency parameter, $\bar{a}$.

AN APPROXIMATE METHOD FOR DETERMINING THE STABILITY OF SPINNING

Now that the stability diagram of the isothermal spinning of convected Maxwell fluids has been obtained using the linear stability analysis method as shown in Fig. 3, the next subject is the approximate method for determining the same stability. This method was developed based on the fundamental phy-

Table 1. The real and imaginary parts of the eigenvalues for the system having $\bar{a}$=0.4 and De=0.02

Draw-down ratio (r)	1st eigenvalue		2nd eigenvalue		3rd eigenvalue	
	Ω_r	Ω_i	Ω_r	Ω_i	Ω_r	Ω_i
15 (stable)	−0.533	12.826	−2.949	31.066	−4.509	49.264
20.237 (critical)	0	13.762	−2.639	33.588	−4.359	53.515
30 (unstable)	0.638	15.119	−2.626	37.492	−4.694	60.235
50 (unstable)	1.131	17.338	−4.147	45.121	−6.925	73.568
70 (unstable)	0.595	19.404	−7.599	54.469	−10.892	89.525
77.811 (critical)	0	20.229	−9.611	59.008	−13.198	97.042
85 (stable)	−0.821	20.995	−11.906	63.805	−15.663	104.824

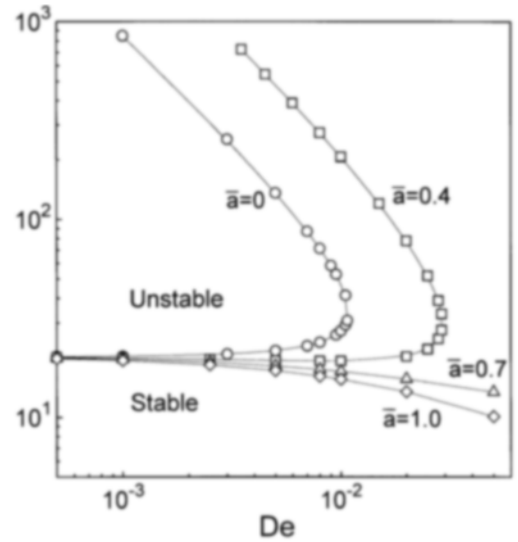

Fig. 3. Stability diagrams of various convected Maxwell fluids by linear stability method.

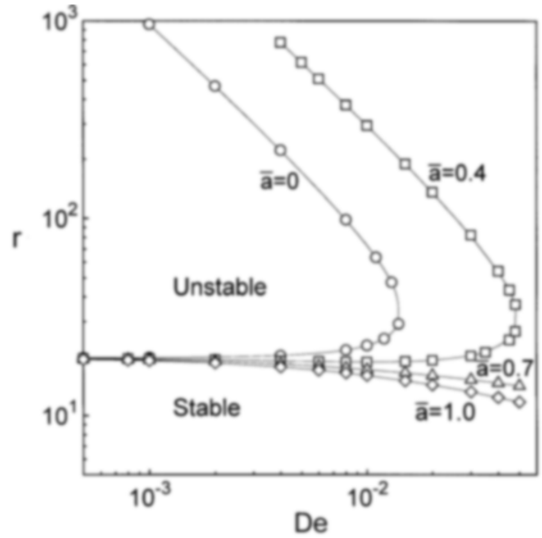

Fig. 4. Approximate stability diagrams corresponding to the same case of Fig. 3.

sics of spinning process, and the details have been reported by Jung et al. [1999] (An approximate method for determining the stability of film casting processes was developed utilizing the same concept of traveling times of throughput waves and the fluid residence time which was previously used to derive the draw resonance criterion in fiber spinning [Kim et al., 1996]). Hence here only the final equation of the method at the onset point of draw resonance is reproduced as follows.

$$2\left(t_L+\frac{\Delta t}{2}\right)\approx\tau_L \quad \text{at} \quad r=r_C \tag{28}$$

where t_L=dimensionless traveling time of throughput waves, Δt= dimensionless time difference between the spinline force and the throughput wave at the take-up, τ_L=dimensionless fluid residence time.

The fluid residence time is obtained from the steady state velocity solution by the following equation.

$$\tau_L=\int_0^1\frac{dx}{v} \tag{29}$$

where v=dimensionless spinline velocity, x=dimensionless spinning distance from the spinneret.

Eq. (28) is further approximated by the following expression which was first used by Hyun [1978].

$$2\left(\frac{\ln r}{r-1}\right)\approx\tau_L \quad \text{at} \quad r=r_C \tag{30}$$

This is the final approximate criterion equation for draw resonance which thus predicts stability or instability of the system according as the left hand side is greater or smaller than the right hand side, respectively. Solving this equation is rather simple because the fluid residence time is always readily obtainable from the steady state velocity solution of the system [Jung et al., 1999].

Fig. 4 shows the stability results thus obtained using Eq. (30). Despite the approximations introduced in the course of deriving this equation, these results agree well with the exact ones of Fig. 3. The utility of the approximate method has thus been demonstrated here. In other words, without having to obtain transient solutions of the spinning equations of Eqs. (8)-(16), the method provides a quick means to determine the stability of the spinning process approximately.

DISCUSSIONS

Now the results of Table 1, Fig. 3 and Fig. 4 are further discussed. First, other than the fact that the data of Table 1 show the way to find the critical draw-down ratio at the onset of draw resonance, one more point is worth mentioning here. The period of the draw resonance oscillation at the onset point is easily obtained from the imaginary part of the eigenvalue as shown below.

$$\text{(imaginary part of the eigenvalue)}=2\pi/T \tag{31}$$

where T=period of draw resonance.

The above relation holds only at the critical draw-down ratio because harmonic oscillations are possible only at the onset of draw resonance, while at higher draw-down ratios the oscillations become skew as shown in Fig. 2.

Table 2 shows the periods of draw resonance at the onset obtained using Eq. (31). These values exactly coincide with those obtained by nonlinear simulations of the system, i.e., transient solutions of the governing equations of Eqs. (8)-(16).

From Fig. 3, the effects of two parameters, i.e., Deborah number, De, and the strain-rate dependency of the relaxation time, $\bar{a}$, on the stability are seen to be interrelated to each other. In other words, depending on whether the parameter $\bar{a}$ is larger or smaller than $1/\sqrt{3}$, the effect of the relaxation time or equivalently Deborah number here, is drastically different. First, if $\bar{a}$ has a value smaller than $1/\sqrt{3}$, there are two stable regions separated by the in-between unstable region, whereas if $\bar{a}$ is larger, only one stable region. Second, if $\bar{a}$ is smaller, there exists a

Table 2. Periods of draw resonance at the onset point for the system having $\bar{a}$=0.4 and varying De

Deborah number (De)	Critical draw-down ratio (r_c)	Imaginary part of the largest eigenvalue (Ω_i)	Period of draw resonance (T)
0 (Newtonian)	20.218	14.008	0.449
0.001	19.951	13.891	0.452
0.005	19.642	13.674	0.459
0.01	19.233	13.627	0.461
0.02	20.237	13.762	0.457
0.02	77.811	20.229	0.311
0.01	207.337	29.411	0.214
0.006	388.692	38.416	0.164

maximum Deborah number beyond which there is no unstable region, whereas if $\bar{a}$ is larger, there is no such critical Deborah number existing. Third, as Deborah number increases, the system becomes less stable when $\bar{a}$ is larger than $1/\sqrt{3}$, but the stability of the system remains unaffected when $\bar{a}$ is smaller.

Assessing what has been described above, we can see that there is a dichotomy of the fluids depending on their stability behavior in spinning, i.e., fluids having smaller values of $\bar{a}$ and those larger values of $\bar{a}$. This dichotomy is not new in that there have been many research results in the last couple of decades reporting the similar differences [Petrie and Denn, 1976; Minoshima and White, 1986; Münstedt and Laun, 1981; Hyun, 1989; Lee et al., 1995]. Such examples include the existence of large vortices in contraction flow by LDPE as opposed to small vortices by HDPE, the multiplicity of flow rates at constant wall shear stress in capillary flow by HDPE as opposed to none by LDPE, the necking behavior by HDPE as opposed to none by LDPE, and large strain-hardening in extensional deformation by LDPE as opposed to a small one by HDPE.

As mentioned above, the agreement between the stability curves in Fig. 3 and Fig. 4 is considered good with rather small numerical discrepancies in spite of the fact the approximate method of Eq. (30) was obtained incorporating a couple of approximations. Since this approximate method only requires a steady state velocity solution to determine the critical draw-down ratio at the onset point of draw resonance, this method is a useful tool to analyze the stability of not only the spinning process but also other extensional deformation processes like film casting [Jung et al., 1999] and film blowing.

CONCLUSIONS

The exact stability curves of isothermal spinning of convected Maxwell fluids have been obtained using the linear stability analysis method. The stable and unstable regions thus depicted in the diagram of the draw-down ratio and Deborah number reveal the effect of the system parameters of the fluid elasticity and strain-rate dependency of the relaxation time on the draw resonance stability. Particularly, the strain-rate dependency of the relaxation time has turned out to dichotomize the viscoelastic fluids into the two groups whose behavior in flow and deformation are distinctly different from each other. An approximate method for determining the stability has also been applied to the same system to produce the stability curves which are close to the exact ones despite the approximations incorporated in the derivation of the method. This approximate method thus is viewed as a useful tool with which to analyze and design extensional deformation processes like film casting and film blowing as well as the spinning processes, from which the method was originally derived.

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NOMENCLATURE

A : spinline cross-sectional area
$\underline{\underline{A}}$: eigenvalue matrix
a : dimensionless spinline cross-sectional area
$\bar{a}$: parameter representing the strain-rate dependency of material relaxation times
De : Deborah number
F : spinline tension force
G : material modulus
L : spinning distance between the spinneret and the take-up
N : number of mesh points in the discretized spinning distance coordinate
r : draw-down ratio
r_C : critical draw-down ratio
T : period of draw resonance
t : dimensionless time
t' : time
t_L : dimensionless traveling time of throughput waves
Δt : dimensionless time difference between the spinline force and the throughput wave at the take-up
V : spinline velocity
v : dimensionless spinline velocity
x : dimensionless distance from the spinneret
$\underline{y}$: eigenvector
z : distance from the spinneret

Greek Letters

α : perturbed quantity related to spinline cross-sectional area
β : perturbed quantity related to spinline velocity
δ : perturbed quantity related to spinline axial stress and radial stress
γ : perturbed quantity related to spinline axial stress
λ : material relaxation time
λ_o : material relaxation time when no strain-rate is applied
σ_{zz} : spinline axial stress
σ_{rr} : spinline radial stress
τ_{xx} : dimensionless spinline axial stress
τ_{rr} : dimensionless spinline radial stress
τ_L : dimensionless fluid residence time
Ω : eigenvalue

Ω_r : real part of eigenvalue
Ω_i : imaginary part of eigenvalue

Superscript

' : differentiation with respect to x, $\partial/\partial x$

Subscripts

0 : values at the spinneret
C : values at critical (onset) point of draw resonance
L : values at the take-up
S : values at steady state

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